**Example**

Suppose sound wave with volume β = 60dB hits water interface. Speed of sound in air is 343 m/s and in water is 1480 m/s. What is volume of reflected sound wave, and transmitted sound wave volume? Treat n as √ρ… (then it works out)



But

**Example: moonlight incident on swimming pool.**

Consider moonlight incident upon a swimming pool, with intensity I0 = 200W/m2. What is the intensity of the reflected light. What is the intensity of the transmitted light? What Erms for the incoming, reflected and transmitted light?

First,



Therefore 98% of the light would be transmitted through water, and 2% of the light would reflected. The intensity of the transmitted light would be:



The rms value of the **E** incident on the water is:



and therefore the **E** that is reflected is:



and the transmitted Erms is:



**Example: moonlight incident on swimming pool.**

How fast is the aforementioned light traveling through the water in the pool? Assuming the incident wavelength is 600nm, what is its wavelength of the light transmitted? What is its frequency? What is the velocity, wavelength, frequency of the reflected light?

The index of refraction of water is 1.33. Therefore the speed of light in water is:



The new wavelength of the light is:



The lights frequency is the same inside or outside of the water. This is:



The velocity of the reflected light is c, the same as the incident light. Its wavelength is 600nm, the same as the incident light. And its frequency is 5×1014Hz, the same as the incident light (since f = c/λ = 3×108/600×10-9 = 5×1014). Since the incident and reflected light are in the same medium, they have the same velocity and wavelength.

4. When red light (λ = 750nm) passes into medium X, its speed slows to v = 1.6×108 m/s. What is the index of refraction of medium X?

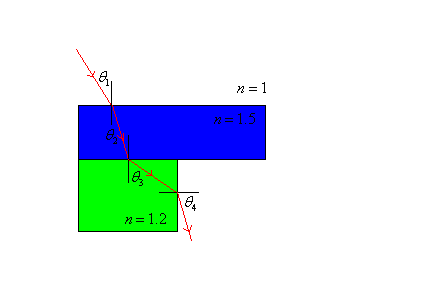


5. If the index of refraction in the problem above were n = 2.3, what would be the light wave’s frequency in medium X.



**Example**

Suppose the angle of incidence is 50 deg. What is the angle of refraction through the first material, θ2? What is the angle through the second material θ3? What is the final exiting angle, θ4? What is the velocity of light in each of the media?



We can use Snell’s law to determine θ2 given θ1 = 50. We have,



As we would expect, θ2 < 50 deg. since the light enters a material with higher index of refraction (going from 1 → 1.5). The angle that the ray makes in the green substance can be determined similarly. In this case, the angle of incidence is also θ2 = 30.7 deg. And so we have,



and as we would expect since it is going from n = 1.5 → 1.2, it bends further away from the normal. Finally, the exiting angle, θ4, can be determined. Note that the angle of incidence 90 – θ3 = 50 deg., and so we have,

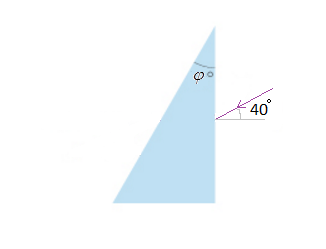


The velocity of light in a medium with index of refraction n is v = c/n. So the velocity in the first medium is:

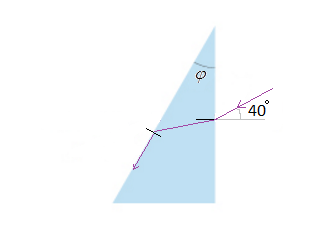


when it exits the media and goes back into air, its velocity will go back to c, since n = 1 for air.

**Question 8**. Consider light incident on the prism with index of refraction 1.67. What angle φ, must the prism face have, to create total internal reflection on the left face?



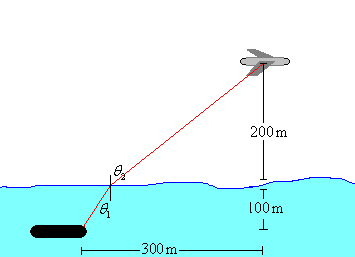
Well we need the path of light to follow:



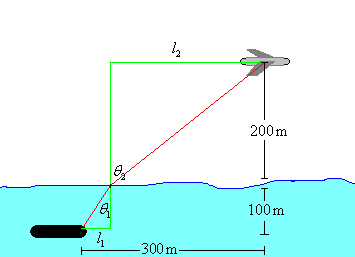
The angle of refraction will be given by 1∙sin(40) = 1.67∙sin(θr) 🡪 θr = 22.7°. Working backwards we have the angle of incidence on the left face is θc = sin-1(n2/n1) = sin(1/1.67) = 36.8°. And so then from geometry we have φ + (90+22.7) + (90-36.8) = 180 🡪 φ = 14.1°.

**Example**

Suppose in submarine and want to fire on airplane using laser. If sub is 100m below water, and airplane is 200m above and 300m to right, at what angle should the laser be fired to hit the airplane?



We’re looking for θ1. Let’s draw in green the following green triangles, and label the horizontal sides to help with the analysis,



Since the index of refraction of water is 1.33, we have,



another relationship between θ1 and θ2 is the following. We know that ℓ1 + ℓ2 = 300m, and we can write ℓ1 and ℓ2 in terms of the tangents of θ1 and θ2. We’ve got,



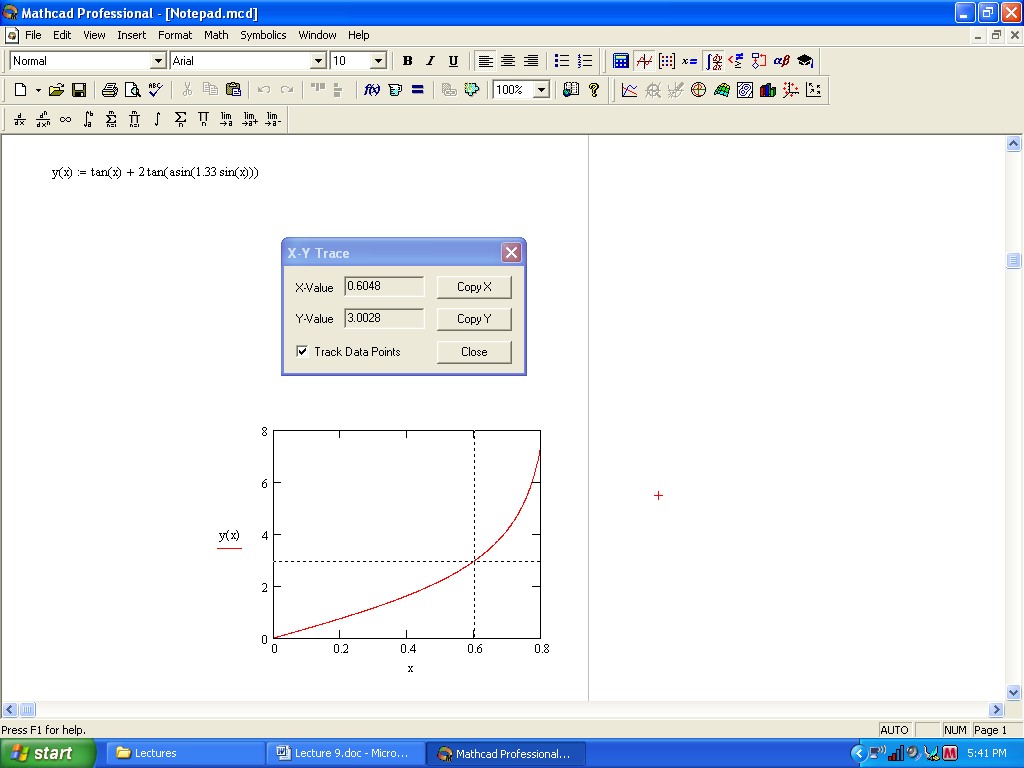
So we have these two simultaneous equations for θ1 and θ2. And we’ll do the usual procedure of solving simultaneous equations - plugging the first one into the second. So plug the Snell equation into the second equation,



How do we solve this? I’m not sure. This situation teaches us a very important lesson. It is frequently the case that we cannot solve equations exactly. But we can do it numerically at least. Define:



and plot it. See at what value of θ the function attains the value of 3. You can use your calculators for this. This is one of their very useful utilities.

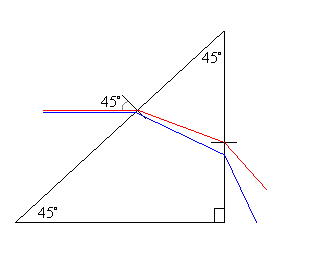


Having plotted it, we see that y(x) = 3 when x = 0.605 rad. So the angle we should fire the laser beam is:



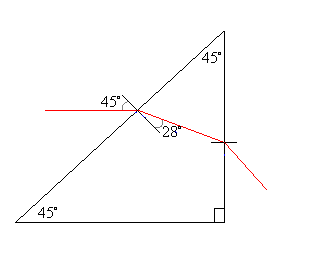
**Example: Prism**

Consider white light incident on a glass prism. The index of refraction of red light in the prism is 1.51, and the index of refraction of blue light in the glass prism is 1.53. Given this what will be the difference in exit angles of the two rays in the following set up??

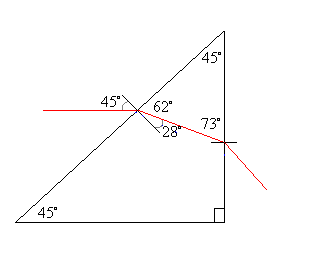


First consider the red light. The angle of incidence is 45 deg. And so the angle of refraction is:



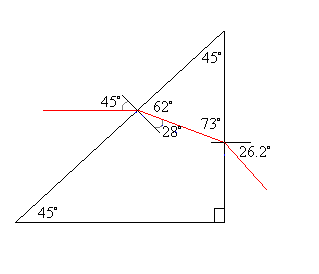


By filling in the angles in the triangle defined by the red line and upper right hand corner of the triangle, we can determine the angle of incidence of the red light on the vertical face.



Therefore the angle of the incidence is 90 – 73 = 17 deg. And using Snell’s law, we have the exit angle,





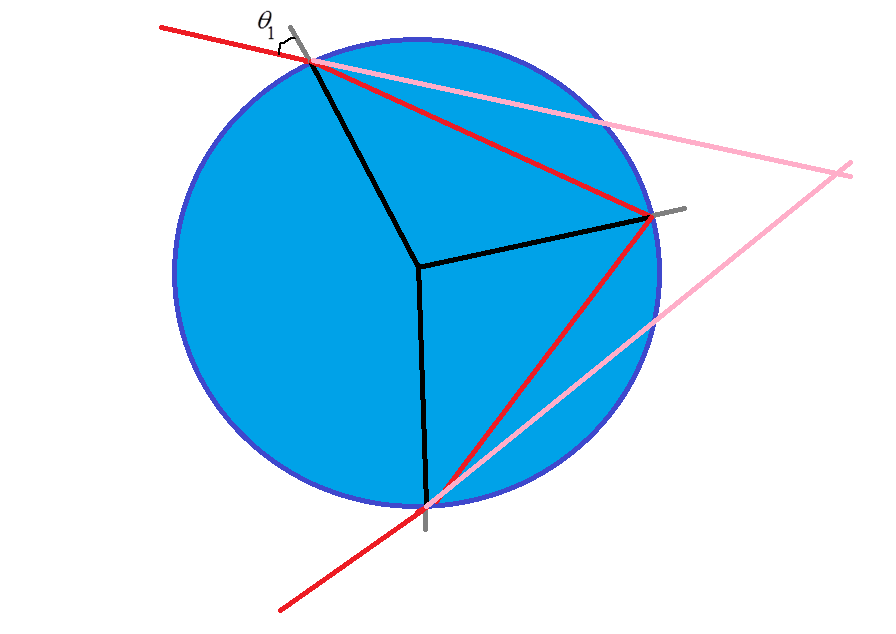
If we repeat the procedure for the blue light, we get an exit angle, θ3, of 27.4 deg. And so the light different colors will be separated by an angle of 27.4 – 26.2 = 1.2 degrees. This is how a prism disperses light.

**Example**

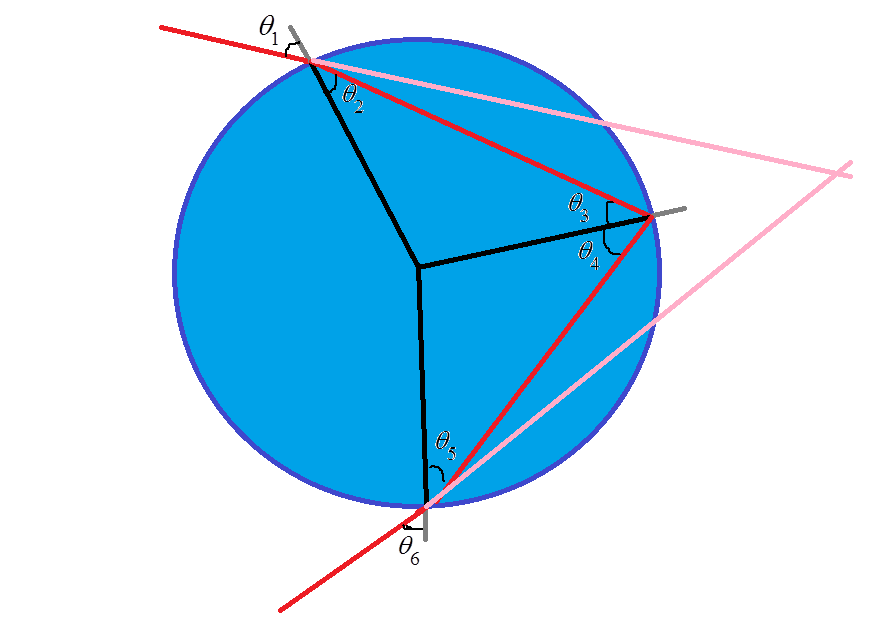
Do total internal reflection of sound over a pool. Can figure out ratio of n’s via ratio of velocities. Use vwater = 1000m/s or something.

**Rainbow**

Suppose have light rays hitting rain drop – draw them at certain angle of declination w/r to the horizontal. They can hit anywhere on the drop, but suppose that they at such a spot that the angle of incidence is 30°. Draw path of light ray through water droplet. And get something like this. Leave out pink lines for now I suppose.

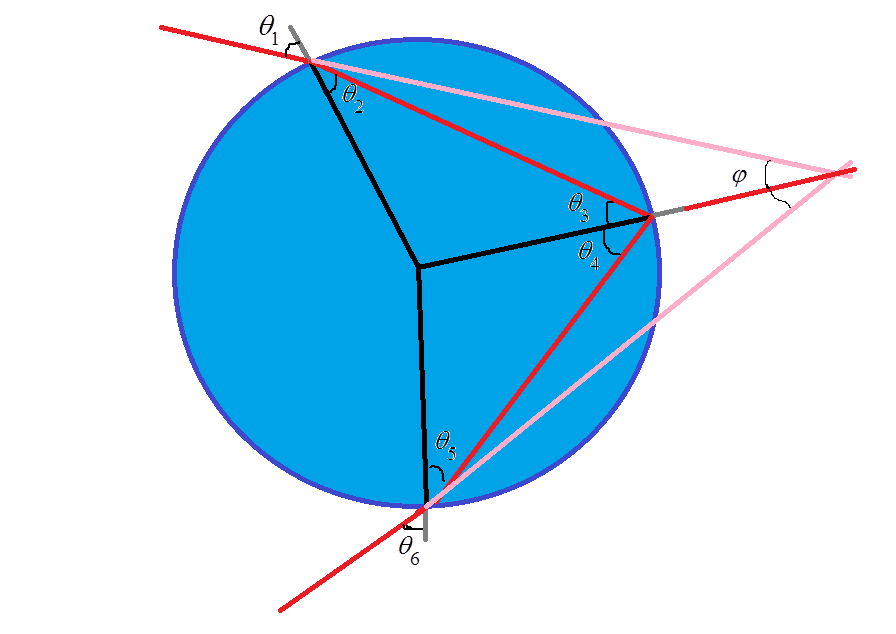


Then take the incident angle to be θ1 = 30. What would be exit angle of reflected ray? Leave all work on board:



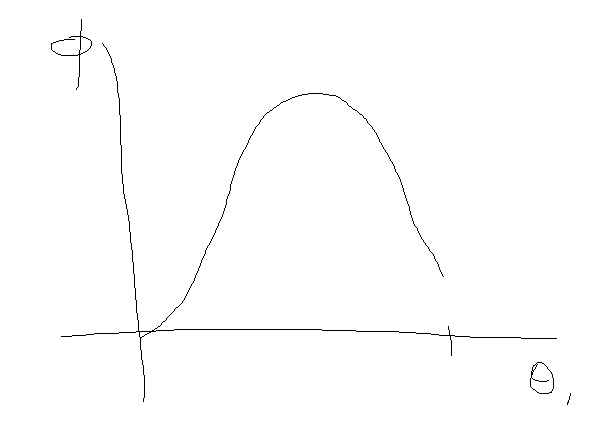


That makes θ3, θ4, and θ5 = 22.1°, and so then θ6 = 30° as well. Next want to know what that pink angle φ is….



And can see φ/2 + θ1 + (180 - 2θ2) = 180 🡪 φ/2 = 2θ2 – θ1 🡪 φ = 4θ2 - 2θ1 = 4(22.1) – 2(30) = 28.4°.

But now let’s generalize to arbitrary θ1. Then φ = 4sin-1(sin(θ1)/1.33) - 2θ1. Do plot of this function:



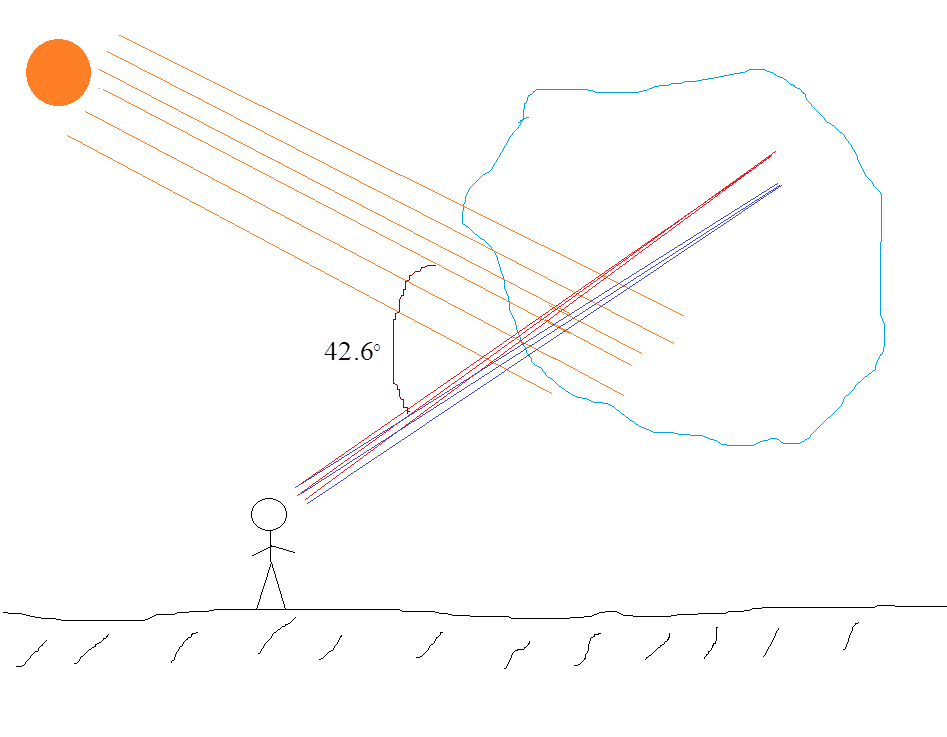
Can see that there is an angle, θ, where the exit angles φ are congregated around. This is where derivative of φ w/r to θ1 = 0. So let’s find it:



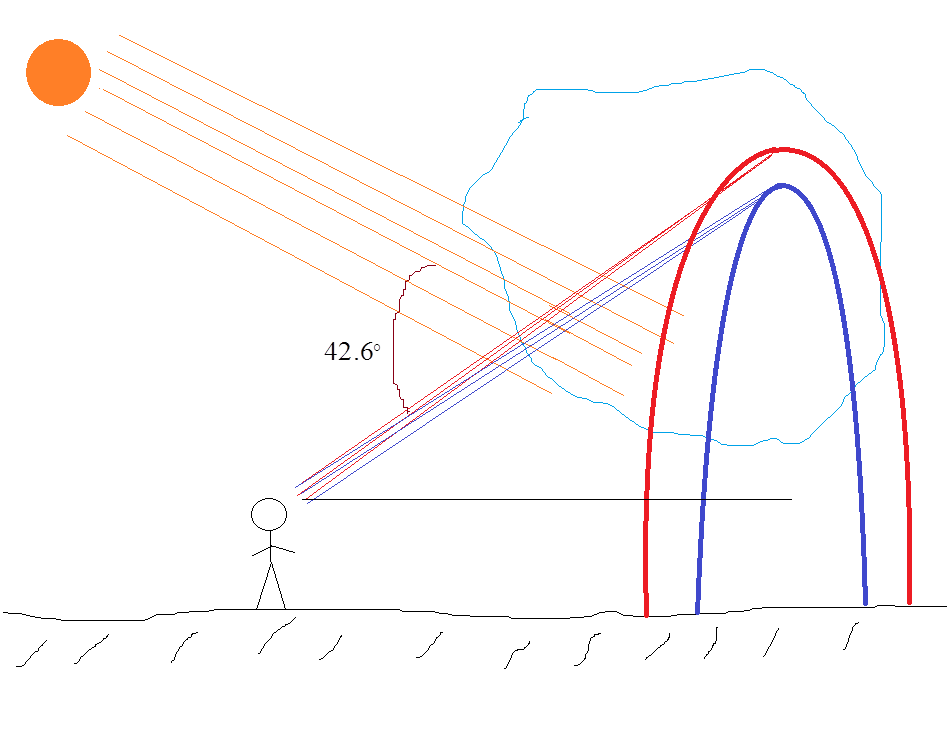
Plugging this back into our equation for φ, we get:



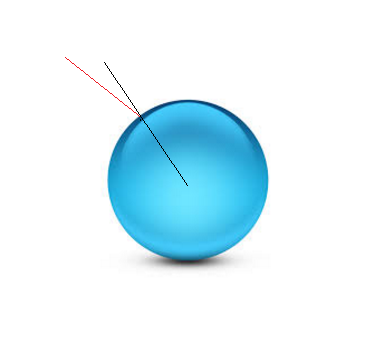
So this will be the angle at which the most intense light will be reflected. Of course you can only form an image of rays that converge at a point and so I would say that you’re seeing rays that are clustered around 42.6° on either side and so which converge. And then the blue light would have a smaller angle, since n is larger for blue light 🡪 40.5° or so. Again, the rays that you see would be clustered on either side of 40.5°.



But note that this image will be reproduced at all angles rotated about the black line, with red rays always making a 42.6 angle and blue ones a 40.5 angle.

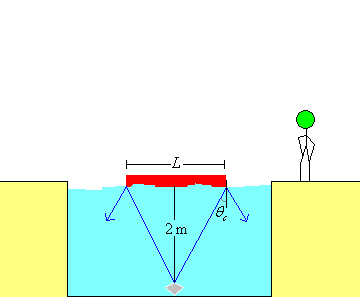


Note the rest of the rainbow gets produced by rays hitting the raindrops ‘off-center’. The plane of refraction will be the plane containing the incident ray and the ray going from the center of the sphere to the point of incidence. And since the sunlight will produce incident rays covering the entire face of the raindrop, you will get exident rays at 42.6° in all planes. Tried to draw this below, but failed.



**Example**

Suppose you want to hide a diamond at the bottom of a 2m deep swimming pool by placing a square raft over it. What is the necessary dimension of the raft?



In order to succesfully hide the diamond, you need to prevent all light that leaves the diamond from leaving the water and entering the air, where the person standing by the side of the pool can see it. But you don’t need to cover the entire pool because light that strikes the water-air interface at angles greater than the critical angle with be automatically reflected back into the water.

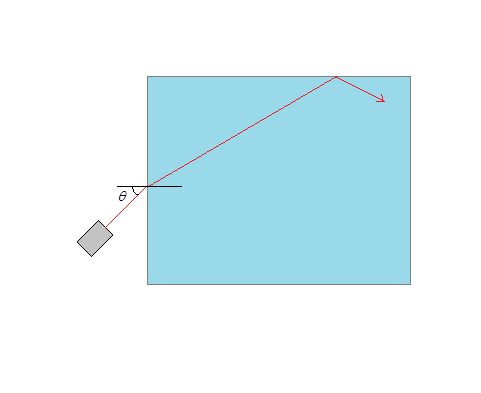
Let us first determine the critical angle,



So we need the raft to be large enough to cover all light striking the water at smaller angles. This requires, by geometry,



6. A laser is aimed at the side of a rectangle of water (n = 1.33). What maximum angle θ will result in total internal reflection on the top side?



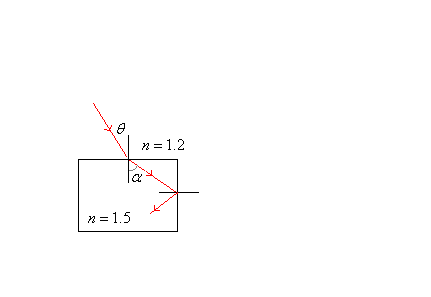
At the top we’ll have:



This would make the angle of refraction on the left side, θ2 = 41°. So then applying Snell’s law again,



3. What maximum angle θ will result in total internal reflection on the vertical face?.



Angle of refraction at the first interface is:



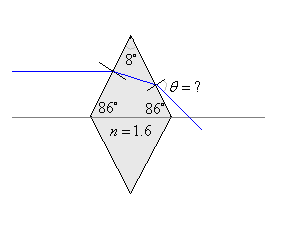
At second interface:



Filling this into the top one:



We have a light beam incident on the prism below. What will be the exit angle?



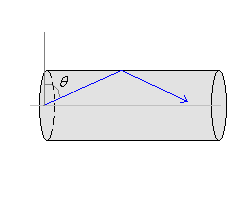
The angle of incidence is 4˚. So the first angle of refraction is:



From this we can see that the refracted ray makes an angle of 90˚ - 2.5˚ = 87.5˚ with the left surface of the prism. And from this we have that the refracted ray makes an angle of 180˚ - 87.5˚ - 8˚ = 84.5˚ with the right hand surface of the prism. Therefore it makes an angle θ3 = 90˚ - 84.5˚ = 5.5˚ with the normal of the right hand side. So then finally, by Snell’s law we can calculate the exit angle. It is:



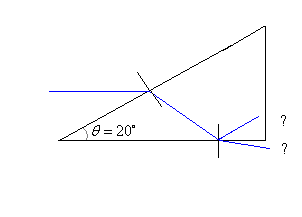
7. Suppose a fiber optic cable (n = 2.3) is surrounded by air. At what angle minimum angle, θ, should a laser light be pointed so that when it hits the surface of the fiber optic cable, the light ray reflects off the surface instead of refracting? C



The angle of incidence at the interface is also θ. In order fo the ray to totally internally reflect off the interface we need,



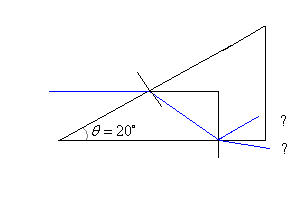
14. Light travels through the following triangle (n = 1.5). At what angle with respect to the horizontal does it emerge? Or does it totally internally reflect?



The incident angle is: 70 deg. Applying Snell’s law,



Now draw triangle.

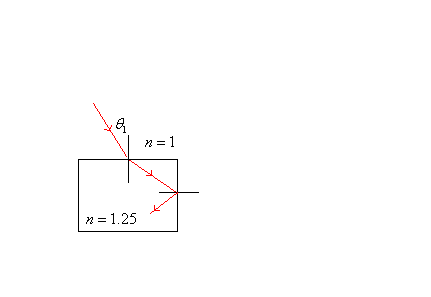


Now 31.1 deg. + θ + 20 = 90 → θ = 38.9, and then that makes incident angle 90 – 38.9 = 51.1 deg. But this is greater than the critical angle,

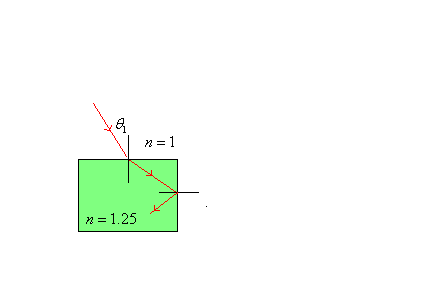


so we get total internal reflection.

9. Consider a different set up. What maximum angle θ1 will result in total internal reflection on the vertical face?.



We have:





for the vertical interface,



Therefore,



And plugging this into the first expression,

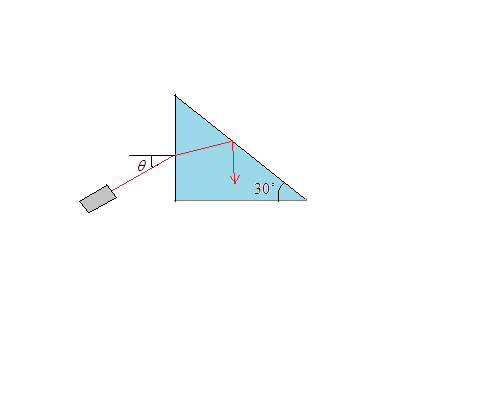




**Example**

At what angles would voice (sound wave) not penetrate water due to total internal reflection off of air-water interface: vair = 346m/s, and vsound = 1480m/s.

6. A laser is aimed at the side of a rectangle of water (n = 1.33). What maximum angle θ will result in total internal reflection on the top side?



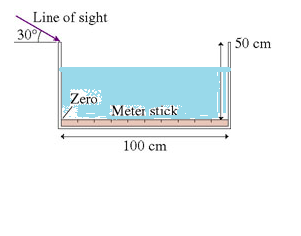
At the top we’ll have:



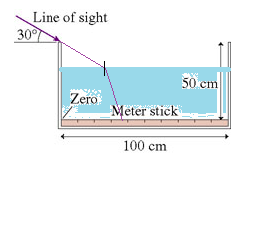
This makes the angle the ray makes with the top side of the triangle 90 – 49 = 41. And this makes the angle the ray forms with the vertical side 180 – 41 – 60 = 79. And this makes the angle it forms w/r to the normal 90 – 79 = 11. And so finally, the incident angle must be:



**Question 5.** A pool of water is 2/3 full. Along the 30° line of sight, what number on the meter stick will you see? Index of refraction of water is 1.33.



Situation looks like this:



Angle of refraction is: n1sinθ1 = n2sinθ2 → θ2 = sin-1[(n1/n2)sinθ1] = sin-1[(1/1.33)sin(60)] = 40.6°. It will strike the pool a distance Δx = Δytan(60) = (1/3∙50)tan(60) = 28.9cm from the left. And so it will hit the bottom at a distance Δx = Δytan(40.6) = (2/3∙50)tan(40.6) = 28.6cm from the interface. So the total distance from the left will be Δx = 28.9cm + 28.6cm = 57.5cm.

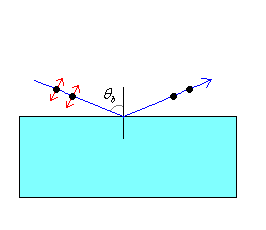
**Question 9**. The glass core of an optical fiber has an index of refraction 1.8. The index of refraction of the cladding is 1.22. What is the maximum angle a light ray can make with the wall of the core if it is to remain inside the fiber?



Polarization

**Example**

What what angle of incidence will light incident upon water be reflected plane polarized?

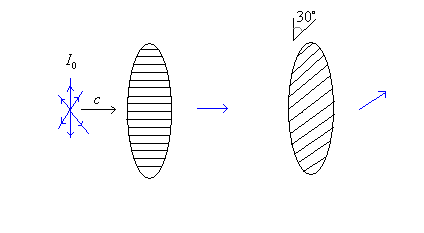


Using the index of refraction of water, the angle would be:



**Example**

Consider randomly polarized light passing through the following setup. What is the final intensity of light if the initial intensity is I0 = 1000W/m2? What is the rms of the Electric field?



The intensity of light making it through the first polarizer is ½ the initial intensity: So



This light is horizontally polarized. And it will make angle of 60 deg. with respect to the polarization axis of the second polarizer. Therefore the intensity of light making it through the second polarzier will be:



The rms of the field will be:



10. If you have vertically polarized light of intensity 150W/m2 , and want to reduce its intensity to 100W/m2, what polarizer should use place in the path of the light? Specify its angle with respect to the vertical.

If you have vertically polarized light of intensity 150W/m2 , and want to reduce its intensity to 100W/m2, what polarizer should use place in the path of the light? i.e., specify its angle with respect to the vertical.



so,

